IRREVERSIBLE DYNAMICS AS A TOOL TO INCREASE ROBUSTNESS IN QUANTUM CONTROL

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General statement

The irreversible dynamics can be useful
for controlling quantum systems

it can steer all initial states
into the same final state

hence producing controls optimal
simultaneously for all system states.

Concrete problems

- How to physically produce arbitrary pure and mixed density matrices?

- How to realize complete density matrix controllability of quantum systems?

- How to physically produce arbitrary all-to-one Kraus maps?

  [maps $\Phi_{\rho_\mathrm{f}} : \mathbb{C}^{n\times n} \to \mathbb{C}^{n\times n}$ that are completely positive, trace preserving and such that for any density matrix ρ: $\Phi_{\rho_\mathrm{f}}(\rho) = \rho_\mathrm{f}$].

Motivation

- Fundamental interest:
  Finding the limits of our ability
  to manipulate quantum systems.

- Possible practical application:
  Quantum computing with mixed states
  (Aharonov, Kitaev, Nisan, Tarasov, Zoller, etc.).

Quantum control: outline

Active field of modern research with ≈ 1300 papers per year.

Applications:

- selective atomic or molecular excitations
- laser-assisted control of chemical reactions
- laser-induced molecular alignment
- quantum gate generation for quantum computing
- and many other applications

Coherent quantum control

- Dynamics: Schrödinger equation

$$\frac{dU_t}{dt} = -i[H_0 + V\varepsilon(t)]U_t, \qquad \rho_T(\varepsilon) = U_T \rho_0 U_T^\dagger$$

- Coherent control: tailored laser field $\varepsilon(t)$

- Objective: $J[\varepsilon] = J(U_T(\varepsilon)) \to \max / \min$

Examples of objectives

- Maximize expectation of a desired system observable O

$$J_1[\varepsilon] = \mathrm{Tr}[\rho_T(\varepsilon)O] \to \max$$

- Prepare a desired state ρ_{target}

$$J_2[\varepsilon] = \|\rho_T(\varepsilon) - \rho_{\mathrm{target}}\| \to \min$$

- Produce a desired unitary gate U_{target}

$$J_3[\varepsilon] = \|U_T(\varepsilon) - U_{\mathrm{target}}\| \to \min$$

Unitary control and reversible dynamics

- The dynamics is unitary and hence reversible.

- Limitation: for $\rho_{\mathrm{i}} \neq \rho'_{\mathrm{i}}$

$$U_T \rho_{\mathrm{i}} U_T^\dagger \neq U_T \rho'_{\mathrm{i}} U_T^\dagger$$

  Optimal controls are usually different for different initial states:

  unitary control is "not robust" to variations of the initial system state.

Open systems: outline

- Open systems are those that interact with their environments.

- Dynamics: master equation

$$\frac{d\rho_t}{dt} = -i[H_0, \rho_t] + \mathcal{L}(\rho_t), \qquad \rho_0 = \rho_{\mathrm{i}}$$

  - Weak couping limit: Bogoliubov, van Hove, Spohn, Lebowitz, Accardi, Lu, Volovich, etc.
  - Low density limit: Dümcke, Spohn, Alicki, Accardi, Lu, Pechen, Volovich.

- Existence under some conditions of a stationary state ρ_{st} such that for all ρ_{i},

$$\rho_{\mathrm{i}} \to \rho_t \to \rho_{\mathrm{st}}$$

Such dynamics is irreversible.

Weak coupling limit

- Hamiltonian $H_\lambda = H_0 + \lambda Q \otimes (a_f + a_f^+)$
- Evolution $U_\lambda(t) = e^{itH_0} e^{-itH}$
- Weak coupling limit:

$$\lambda \to 0, \qquad t \to +\infty, \qquad \lambda^2 t = \tau = \mathrm{const}$$

System dynamics: $\rho_\tau = \lim_{\lambda \to 0} \mathrm{Tr}_{\mathrm{E}}[U_\lambda(t)(\rho_0 \otimes \omega_{\mathrm{E}}) U_\lambda^\dagger(t)]$

Total dynamics: $U_\tau = \lim_{\lambda \to 0} U_\lambda(\tau/\lambda^2)$

- Master equation: many derivations.
- QSDE for U_τ: Accardi, Lu, Volovich (Book "Quantum theory and its stochastic limit", 2002).

Low density limit

- Hamiltonian $H = H_0 + Q \otimes a_f^+ a_g + Q^\dagger \otimes a_g^+ a_f$
- Gaussian environment

$$\omega_{\mathrm{E}}(a_k^+ a_{k'}) = \epsilon n_k \delta(k - k')$$

- Low density limit:

$$\epsilon \to 0, \qquad t \to +\infty, \qquad nt = \tau = \mathrm{const}$$

- Master equation: Dümcke using BBGKY hierarchy (1984)
- QSDE for U_τ: Accardi, Lu, Alicki, Pechen, Volovich

Incoherent control

- Master equation

$$\frac{d\rho_t}{dt} = -i[H_0 + Vu(t), \rho_t] + \mathcal{L}_{u,n_\omega}(\rho_t)$$

- Controls:
  - Coherent light with intensity $u(t)$.
  - Incoherent environment with spectral density n_ω.

Ref.: Pechen, Rabitz, Phys. Rev. A **73**, 062102 (2006)

Control by incoherent light

- Superoperator produced by incoherent light:

$$\mathcal{L}_{n_\omega}(\rho) = \sum_{i<j} A_{ij}[(n_{\omega_{ij}}+1)L_{Q_{ij}}(\rho) + n_{\omega_{ij}} L_{Q_{ji}}(\rho)]$$

$$L_Q(\rho) = 2Q\rho Q^\dagger - Q^\dagger Q\rho - \rho Q^\dagger Q,$$

$$Q_{ij} = |i\rangle\langle j|, \qquad A_{ij} \geq 0$$

- Control: spectral density of incoherent light $n_\omega \geq 0$.

Main result: Engineering non-degenerate density matrices

The irreversible dynamics can be used to produce all non-degenerate states $\rho_{\mathrm{f}} = \sum p_i |\phi_i\rangle\langle\phi_i|$ via the following scheme:

1. Incoherent stage. Using special n_ω, create the mixed state $\tilde{\rho}_{\mathrm{f}} = \sum p_i |i\rangle\langle i|$ diagonal in the basis of H_0 (find such n_ω that $\mathcal{L}_{n_\omega}$ has $\tilde{\rho}_{\mathrm{f}}$ as the stationary state). The state evolves as $\rho_{\mathrm{i}} \to \rho_t \to \tilde{\rho}_{\mathrm{f}}$.

2. Coherent stage. Using special $u(t)$, create a unitary evolution transforming $\tilde{\rho}_{\mathrm{f}} \to \rho_{\mathrm{f}}$ (standard coherent control).

This control scheme evolves all initial states into the same ρ_{f}.

Ref: Pechen, Engineering arbitrary pure and mixed quantum states, Phys. Rev. A **84**, 042106 (2011)

Property 1: Complete density matrix controllability

- 1\. Pure state controllability:
  $|\psi_{\rm i}\rangle \to |\psi_{\rm f}\rangle$; $|\psi_{\rm i}\rangle$ and $|\psi_{\rm f}\rangle$ are any pure states
- 2\. Density matrix controllability:
  $\rho_{\rm i} \to \rho_{\rm f}$; $\rho_{\rm i}$ and $\rho_{\rm f}$ have the same spectrum
- 3\. Complete density matrix controllability (CDMC):
  $\rho_{\rm i} \to \rho_{\rm f}$; $\rho_{\rm i}$ and $\rho_{\rm f}$ are any density matrices:
  the strongest degree of quantum state control:
  $3 \Rightarrow 1, 2$

Result: A combination of the irreversible incoherent control and the reversible coherent control can approximately realize complete density matrix controllability for a wide class of quantum systems.

Property 2: Production of all-to-one Kraus maps

Definition. A Kraus map is a completely positive trace preserving linear map $\Phi : \mathbb{C}^{n\times n} \to \mathbb{C}^{n\times n}$.

Definition. An all-to-one Kraus map Φ_{ρ_f} is a Kraus map such that $\Phi_{\rho_\mathrm{f}}(\rho_\mathrm{i}) = \rho_\mathrm{f}$ for all ρ_i.

Ref: Wu, Pechen, Brif, Rabitz, J. Phys. A **40**, 5681 (2007)

Result: The combination of incoherent and coherent controls n_ω and $\varepsilon(t)$ can approximately produce any all-to-one Kraus map Φ_{ρ_f}.

Ref: Pechen, *Engineering arbitrary pure and mixed quantum states*, Phys. Rev. A **84**, 042106 (2011)

Example: Calcium atom

$$\rho_t = \frac{1}{2}\left[\mathbb{I} + r_x(t)\sigma_x + r_y(t)\sigma_y + r_z(t)\sigma_x\right]$$

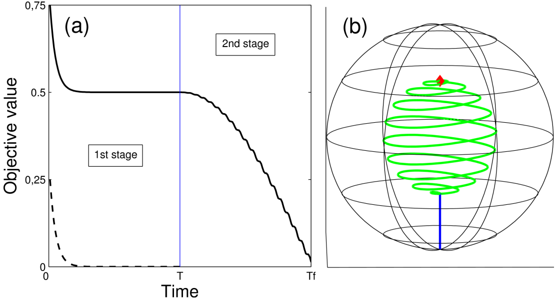

Figure: (From Phys. Rev. A **84**, 042106 (2011). Copyright by the American Physical Society, 2011.) Dash line is for $\|\rho_t - \tilde{\rho}_{\mathrm{f}}\|$ and solid line for $\|\rho_t - \rho_{\mathrm{f}}\|$.

Conclusions

Irreversible dynamics can be used for a robust engineering of pure and mixed quantum states.

Properties:

- Robustness to variations of the initial state
- Realization of the complete density matrix controllability
- Production of all-to-one Kraus maps

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